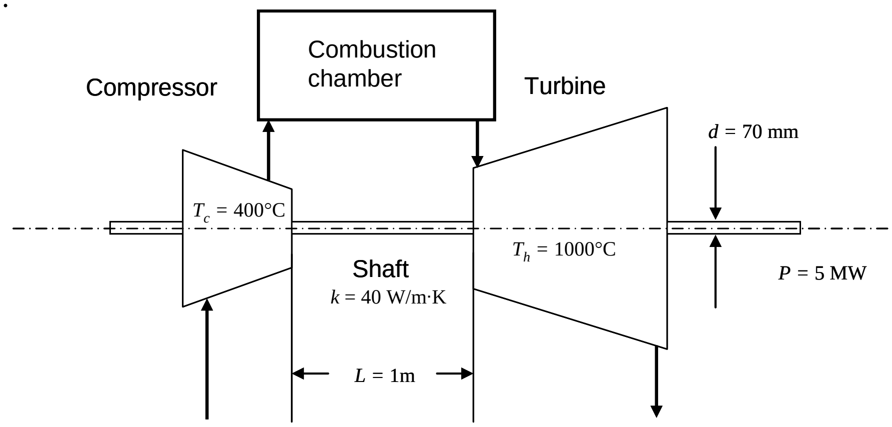
PROBLEM 1.8

KNOWN: Net power output, average compressor and turbine temperatures, shaft dimensions and thermal conductivity.

FIND: (a) Comparison of the conduction rate through the shaft to the predicted net power output of the device, (b) Plot of the ratio of the shaft conduction heat rate to the anticipated net power output of the device over the range $0.005 \text{ m} \le L \le 1 \text{ m}$ and feasibility of a L = 0.005 m device.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) Net power output is proportional to the volume of the gas turbine.

PROPERTIES: Shaft (given): k = 40 W/m·K.

ANALYSIS: (a) The conduction through the shaft may be evaluated using Fourier's law, yielding

$$q = q''A_c = \frac{k(T_h - T_c)}{L}\left(\pi d^2/4\right) = \frac{40\text{W/m}\cdot\text{K}(1000-400)°\text{C}}{1\text{m}}\left(\pi(70\times10^{-3}\text{m})^2/4\right) = 92.4\text{W}$$

The ratio of the conduction heat rate to the net power output is

$$r = \frac{q}{P} = \frac{92.4\text{W}}{5\times10^6\text{W}} = 18.5\times10^{-6}$$ <

(b) The volume of the turbine is proportional to L^3. Designating L_a = 1 m, d_a = 70 mm and P_a as the shaft length, shaft diameter, and net power output, respectively, in part (a),

$$d = d_a \times (L/L_a);\ P = P_a \times (L/L_a)^3$$

and the ratio of the conduction heat rate to the net power output is

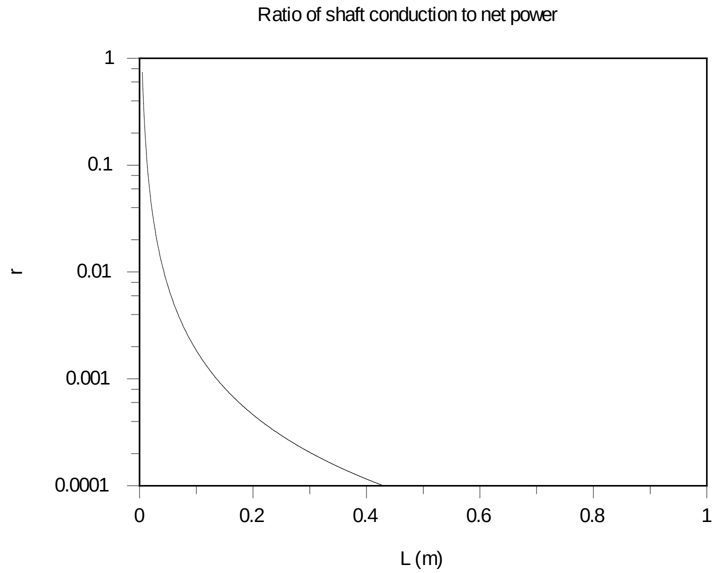
$$r = \frac{q''A_c}{P} = \frac{\frac{k(T_h - T_c)}{L}\left(\pi d^2/4\right)}{P} = \frac{\frac{k(T_h - T_c)}{L}\left(\pi\left(d_a L/L_a\right)^2/4\right)}{P_a(L/L_a)^3} = \frac{\frac{k(T_h - T_c)\pi}{4} d_a^2 L_a/P_a}{L^2}$$

$$= \frac{\frac{40\text{W/m}\cdot\text{K}(1000-400)°\text{C}\pi}{4}(70\times10^{-3}\text{m})^2\times1\text{m}/5\times10^6\text{W}}{L^2} = \frac{18.5\times10^{-6}\text{m}^2}{L^2}$$

Continued…

PROBLEM 1.8 (Cont.)

The ratio of the shaft conduction to net power is shown below. At L = 0.005 m = 5 mm, the shaft conduction to net power output ratio is 0.74. The concept of the very small turbine is not feasible since it will be unlikely that the large temperature difference between the compressor and turbine can be maintained. <



COMMENTS: (1) The thermodynamics analysis does not account for heat transfer effects and is therefore meaningful only when heat transfer can be safely ignored, as is the case for the shaft in part (a). (2) Successful miniaturization of thermal devices is often hindered by heat transfer effects that must be overcome with innovative design.